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Minimal *rp*-open Sets and Maximal *rp*-open Sets

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Abstract

The object of the present paper is to study the notions of minimal rp-open set and maximal *rp*-open set and their basic properties are studied.

Keywords: *rp*-open set, minimal *rp*-open set and maximal *rp*-open set.

Introduction

Nakaoka and Oda have introduced minimal open sets and maximal open sets, which are subclasses of open sets. A. Vadivel and K. Vairamanickam introduced minimal rgα-open sets and maximal rgα-open sets in topological spaces. S. Balasubramanian and P.A.S. Vyjayanthi introduced minimal v-open sets and maximal v-open sets; minimal v-closed sets and maximal v-closed sets in topological spaces. Inspired with these developments we further study a new type of open sets namely minimal rp-open sets and maximal rp-open sets.

Throughout the paper a space X means a topological space (X, τ). The class of rp-open sets is denoted by RPO(X). For any subset A of X its complement, interior, closure, rp-interior, rp-closure are denoted respectively by the symbols A^c, A^o, A^-, $rp(A)^0$ and $rp(A)^-$.

Preliminaries

Definition 1: A proper nonempty

(i) open subset U of X is said to be a **minimal open set** if any open set contained in U is ϕ or U.

(ii) semi-open subset U of X is said to be a **minimal semi-open set** if any semi-open set contained in U is ϕ or U.

(iii) pre-open subset U of X is said to be a **minimal pre-open set** if any pre-open set contained in U is ϕ or U.

(iv) *v*-open subset U of X is said to be a **minimal *v*-open set** if any *v*-open set contained in U is ϕ or U.

(v) *rgα*-open subset U of X is said to be a **minimal *rgα*-open set** if any *rgα*-open set contained in U is ϕ or U.

Definition 2: A proper nonempty

(i) open subset U of X is said to be a **maximal open set** if any open set containing U is X or U.

(ii) semi-open subset U of X is said to be a **maximal semi-open set** if any semi-open set containing U is X or U.

(iii) pre-open subset U of X is said to be a **maximal pre-open set** if any pre-open set containing U is X or U.

(iv) *v*-open subset U of X is said to be a **maximal *v*-open set** if any *v*-open set containing U is X or U.

(v) *rgα*-open subset U of X is said to be a **maximal *rgα*-open set** if any *rgα*-open set containing U is X or U.

Minimal *rp*-open Sets and Maximal *rp*-open Sets

Definition 1: A proper nonempty *rp*-open subset U of X is said to be a **minimal *rp*-open set** if any *rp*-open set contained in U is ϕ or U.

Remark 1: Minimal open set and minimal *rp*-open set are not same:

Example 1: Let X = {a, b, c}; τ = {ϕ, {a, c}, X}. {a, c} is Minimal open but not Minimal *rp*-open; {a} and {c} are Minimal *rp*-open but not Minimal open.

Remark 2: From the above example and known results we have the following implications

Theorem 1: (i) Let U be a minimal *rp*-open set and W be a *rp*-open set. Then $U \cap W = \phi$ or $U \subset W$.

(ii) Let U and V be minimal *rp*-open sets. Then $U \cap V = \phi$ or U = V.

Proof: (i) Let U be a minimal *rp*-open set and W be a *rp*-open set. If $U \cap W = \phi$, then there is nothing to prove. If $U \cap W \neq \phi$. Then $U \cap W \subset U$. Since U is a minimal *rp*-open set, we have $U \cap W = U$. Therefore $U \subset W$.

(ii) Let U and V be minimal *rp*-open sets. If $U\cap V \neq \phi$, then $U\subset V$ and $V\subset U$ by (i). Therefore U = V.

Theorem 2: Let U be a minimal *rp*-open set. If $x\in U$, then $U\subset W$ for any regular open neighborhood W of x.

Proof: Let U be a minimal *rp*-open set and x be an element of U. Suppose $\exists$ a regular open neighborhood W of x such that $U \not\subset W$. Then $U \cap W$ is a *rp*-open set such that $U \cap W \subset U$ and $U \cap W \neq \phi$. Since U is a minimal *rp*-open set, we have $U\cap W = U$. That is $U \subset W$, which is a contradiction for $U \not\subset W$. Therefore $U\subset W$ for any regular open neighborhood W of x.

Theorem 3: Let U be a minimal *rp*-open set. If $x\in U$, then $U\subset W$ for some *rp*-open set W containing x.

Theorem 4: Let U be a minimal *rp*-open set. Then $U = \cap\{W: W\in RPO(X, x)\}$ for any element x of U.

Proof: By theorem[3.3] and U is *rp*-open set containing x, we have $U\subset\cap\{ W: W\in RPO(X, x)\}\subset U$.

Theorem 5: Let U be a nonempty *rp*-open set. Then the following three conditions are equivalent.

(i) U is a minimal *rp*-open set

(ii) $U\subset rp(S)^-$ for any nonempty subset S of U

(iii) $rp(U)^- = rp(S)^-$ for any nonempty subset S of U.

Proof: (i) $\Rightarrow$ (ii) Let $x\in U$; U be minimal *rp*-open set and $S(\neq \phi)\subset U$. By theorem[3.3], for any *rp*-open set W containing x, $S\subset U\subset W \Rightarrow S\subset W$. Now $S = S\cap U\subset S\cap W$. Since $S\neq \phi$, $S\cap W \neq \phi$. Since W is any *rp*-open set containing x, by theorem[5.03], $x\in rp(S)^-$. That is $x\in U \Rightarrow x\in rp(S)^- \Rightarrow U\subset rp(S)^-$ for any nonempty subset S of U.

(ii) $\Rightarrow$ (iii) Let S be a nonempty subset of U. That is $S\subset U \Rightarrow rp(S)^-\subset rp(U)^- \rightarrow (1)$. Again from (ii) $U\subset rp(S)^-$ for any $S(\neq \phi)\subset U \Rightarrow rp(U)^-\subset rp(rp(S)^-)^- = rp(S)^-$. That is $rp(U)^-\subset rp(S)^- \rightarrow (2)$. From (1) and (2), we have $rp(U)^- = rp(S)^-$ for any nonempty subset S of U.

(iii) $\Rightarrow$ (i) From (3) we have $rp(U)^- = rp(S)^-$ for any nonempty subset S of U. Suppose U is not a minimal *rp*-open set. Then $\exists$ a nonempty *rp*-open set V such that $V \subset U$ and $V \neq U$. Now $\exists$ an element a in U such that $a\notin V \Rightarrow a\in V^c$. That is $rp(\{a\})^-\subset rp(V^c)^- = V^c$, as V^c is *rp*-closed set in X. It follows that $rp(\{a\})^- \neq rp(U)^-$. This is a contradiction for $rp(\{a\})^- = rp(U)^-$ for any $\{a\}(\neq \phi)\subset U$. Therefore U is a minimal *rp*-open set.

Theorem 6: Let $V \neq \phi$ be a finite *rp*-open set. Then $\exists$ at least one (finite) minimal *rp*-open set U such that $U\subset V$.

Proof: Let V be a nonempty finite *rp*-open set. If V is a minimal *rp*-open set, we may set U = V. If V is not a minimal *rp*-open set, then $\exists$ (finite) *rp*-open set V_1 such that $\phi \neq V_1\subset V$. If V_1 is a minimal *rp*-open set, we may set $U = V_1$. If V_1 is not a minimal *rp*-open set, then $\exists$ (finite) *rp*-open set V_2 such that $\phi \neq V_2 \subset V_1$. Continuing this process, we have a sequence of *rp*-open sets $V \supset V_1 \supset V_2 \supset V_3\supset ..... \supset V_k \supset ......$ Since V is a finite set, this process repeats only finitely. Then finally we get a minimal *rp*-open set $U = V_n$ for some positive integer n.

[A topological space X is said to be locally finite space if each of its elements is contained in a finite open set.]

Corollary 1: Let X be a locally finite space and $V \neq \phi$ be an *rp*-open set. Then $\exists$ at least one (finite) minimal *rp*-open set U such that $U \subset V$.

Proof: Let X be a locally finite space and V be a nonempty *rp*-open set. Let x in V. Since X is locally finite space, we have a finite open set V_x such that x in V_x. Then $V\cap V_x$ is a finite *rp*-open set. By Theorem 3.6 $\exists$ at least one (finite) minimal *rp*-open set U such that $U \subset V\cap V_x$. That is $U\subset V\cap V_x\subset V$. Hence $\exists$ at least one (finite) minimal *rp*-open set U such that $U\subset V$.

Corollary 2: Let V be a finite minimal open set. Then $\exists$ at least one (finite) minimal *rp*-open set U such that $U\subset V$.

Proof: Let V be a finite minimal open set. Then V is a nonempty finite *rp*-open set. By Theorem 3.6, $\exists$ at least one (finite) minimal *rp*-open set U such that $U\subset V$.

Theorem 7: Let U; U_λ be minimal *rp*-open sets for any element $\lambda\in\Gamma$. If $U\subset \cup_{\lambda\in\Gamma}U_\lambda$, then $\exists$ an element $\lambda \in\Gamma$ such that $U = U_\lambda$.

Proof: Let $U \subset \cup_{\lambda\in\Gamma}U_\lambda$. Then $U \cap(\cup_{\lambda\in\Gamma}U_\lambda) = U$. That is $\cup_{\lambda\in\Gamma}(U \cap U_\lambda) = U$. Also by theorem[3.1] (ii), $U \cap U_\lambda = \phi$ or $U = U_\lambda$ for any $\lambda\in\Gamma$. It follows that $\exists$ an element $\lambda\in\Gamma$ such that $U = U_\lambda$.

Theorem 8: Let U; U_λ be minimal *rp*-open sets for any $\lambda\in\Gamma$. If $U = U_\lambda$ for any $\lambda\in\Gamma$, then $(\cup_{\lambda\in\Gamma}U_\lambda) \cap U = \phi$.

Proof: Assume $(\cup_{\lambda\in\Gamma}U_\lambda) \cap U\neq \phi$. That is $\cup_{\lambda\in\Gamma}(U_\lambda \cap U) \neq \phi$. Then $\exists$ an element $\lambda\in\Gamma$ such that $U \cap U_\lambda \neq \phi$. By theorem 3.1(ii), we have $U = U_\lambda$, which contradicts the fact that $U \neq U_\lambda$ for any $\lambda\in\Gamma$. Hence $(\cup_{\lambda\in\Gamma}U_\lambda)\cap U = \phi$.

We now introduce maximal *rp*-open sets in topological spaces as follows.

Definition 2: A proper nonempty *rp*-open $U\subset X$ is said to be **maximal *rp*-open set** if any *rp*-open set containing U is either X or U.

Remark 3: Maximal open set and maximal *rp*-open set are not same.

Example 2: In Example 1, {a, c} is Maximal open but not Maximal *rp*-open; {a, b} and {b, c} are Maximal *rp*-open but not Maximal open.

Remark 4: From the known results and by the above example we have the following implications:

Theorem 9: A proper nonempty subset F of X is maximal *rp*-open set iff X-F is a minimal *rp*-closed set.

Proof: Let F be a maximal *rp*-open set. Suppose X-F is not a minimal *rp*-open set. Then ∃ *rp*-open set U ≠ X-F such that $\phi \neq U \subset X-F$. That is $F \subset X-U$ and X-U is a *rp*-open set which is a contradiction for F is a minimal *rp*-closed set.

Conversely let X-F be a minimal *rp*-open set. Suppose F is not a maximal *rp*-open set. Then ∃ *rp*-open set E ≠ F such that $F \subset E \neq X$. That is $\phi \neq X-E \subset X-F$ and X-E is a *rp*-open set which is a contradiction for X-F is a minimal *rp*-closed set. Therefore F is a maximal *rp*-open set.

Theorem 10: (i) Let F be a maximal *rp*-open set and W be a *rp*-open set. Then $F \cup W = X$ or $W \subset F$.

(ii) Let F and S be maximal *rp*-open sets. Then $F \cup S = X$ or $F = S$.

Proof: (i) Let F be a maximal *rp*-open set and W be a *rp*-open set. If $F \cup W = X$, then there is nothing to prove. Suppose $F \cup W \neq X$. Then $F \subset F \cup W$. Therefore $F \cup W = F \Rightarrow W \subset F$.

(ii) Let F and S be maximal *rp*-open sets. If $F \cup S \neq X$, then we have $F \subset S$ and $S \subset F$ by (i). Therefore F = S.

Theorem 11: Let F be a maximal *rp*-open set. If x is an element of F, then for any *rp*-open set S containing x, $F \cup S = X$ or $S \subset F$.

Proof: Let F be a maximal *rp*-open set and x is an element of F. Suppose ∃ *rp*-open set S containing x such that $F \cup S \neq X$. Then $F \subset F \cup S$ and $F \cup S$ is a *rp*-open set, as the finite union of *rp*-open sets is a *rp*-open set. Since F is a *rp*-open set, we have $F \cup S = F$. Therefore $S \subset F$.

Theorem 12: Let F_α, F_β, F_δ be maximal *rp*-open sets such that $F_\alpha \neq F_\beta$. If $F_\alpha \cap F_\beta \subset F_\delta$, then either $F_\alpha = F_\delta$ or $F_\beta = F_\delta$

Proof: Given that $F_\alpha \cap F_\beta \subset F_\delta$. If $F_\alpha = F_\delta$ then there is nothing to prove.

If $F_\alpha \neq F_\delta$ then we have to prove $F_\beta = F_\delta$. Now $F_\beta \cap F_\delta = F_\beta \cap (F_\delta \cap X) = F_\beta \cap (F_\delta \cap (F_\alpha \cup F_\beta)$(by thm. 3.10 (ii)) $= F_\beta \cap ((F_\delta \cap F_\alpha) \cup (F_\delta \cap F_\beta)) = (F_\beta \cap F_\delta \cap F_\alpha) \cup (F_\beta \cap F_\delta \cap F_\beta)$

$= (F_\alpha \cap F_\beta) \cup (F_\delta \cap F_\beta)$ (by $F_\alpha \cap F_\beta \subset F_\delta$) $= (F_\alpha \cup F_\delta) \cap F_\beta = X \cap F_\beta$ (Since F_α and F_δ are maximal *rp*-open sets by theorem[3.10](ii), $F_\alpha \cup F_\delta = X$) $= F_\beta$. That is $F_\beta \cap F_\delta = F_\beta \Rightarrow F_\beta \subset F_\delta$ Since F_β and F_δ are maximal *rp*-open sets, we have $F_\beta = F_\delta$ Therefore $F_\beta = F_\delta$

Theorem 13: Let F_α, F_β and F_δ be different maximal *rp*-open sets to each other. Then $(F_\alpha \cap F_\beta) \not\subset (F_\alpha \cap F_\delta)$.

Proof: Let $(F_\alpha \cap F_\beta) \subset (F_\alpha \cap F_\delta) \Rightarrow (F_\alpha \cap F_\beta) \cup (F_\delta \cap F_\beta) \subset (F_\alpha \cap F_\delta) \cup (F_\delta \cap F_\beta) \Rightarrow (F_\alpha \cup F_\delta) \cap F_\beta \subset F_\delta \cap (F_\alpha \cup F_\beta)$. Since by theorem 3.10(ii), $F_\alpha \cup F_\delta = X$ and $F_\alpha \cup F_\beta = X \Rightarrow X \cap F_\beta \subset F_\delta \cap X \Rightarrow F_\beta \subset F_\delta$ From the definition of maximal *rp*-open set it follows that $F_\beta = F_\delta$, which is a contradiction to the fact that F_α, F_β and F_δ are different to each other. Therefore $(F_\alpha \cap F_\beta) \not\subset (F_\alpha \cap F_\delta)$.

Theorem 14: Let F be a maximal *rp*-open set and x be an element of F. Then $F = \cup$ { S: S is a *rp*-open set containing x such that $F \cup S \neq X$}.

Proof: By theorem 3.12 and fact that F is a *rp*-open set containing x, we have $F \subset \cup${ S: S is a *rp*-open set containing x such that $F \cup S \neq X$} – F. Therefore we have the result.

Theorem 15: If $F \neq \phi$ is proper cofinite *rp*-open set. Then ∃ (cofinite) maximal *rp*-open set E such that $F \subset E$.

Proof: If F is maximal *rp*-open set, we may set E = F. If F is not a maximal *rp*-open set, then ∃ (cofinite) *rp*-open set F_1 such that $F \subset F_1 \neq X$. If F_1 is a maximal *rp*-open set, we may set $E = F_1$. If F_1 is not a maximal *rp*-open set, then ∃ a (cofinite) *rp*-open set F_2 such that $F \subset F_1 \subset F_2 \neq X$. Continuing this process, we have a sequence of *rp*-open, $F \subset F_1 \subset F_2 \subset \ldots \subset F_k \subset \ldots$. Since F is a cofinite set, this process repeats only finitely. Then, finally we get a maximal *rp*-open set $E = E_n$ for some positive integer n.

Theorem 16: Let F be a maximal *rp*-open set. If $x \in X-F$. Then $X-F \subset E$ for any *rp*-open set E containing x.

Proof: Let F be a maximal *rp*-open set and x in X-F. $E \not\subset F$ for any *rp*-open set E containing x. Then $E \cup F = X$ by theorem 3.10(ii). Therefore $X-F \subset E$.

Conclusion

In this paper we introduced the concept of minimal *rp*-open and maximal *rp*-open sets, studied their basic properties.

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